Kuwait University | Math. 101 | October 31, 2002
Dept. of Math. & Comp. Sci. | First Exam. | Duration: 75 minutes

Calculators, Mobile Phones and Pagers are not allowed.

Answer the following questions.

1. Find the vertical and horizontal asymptotes for the graph of f, (if any) where

$$f(x)=\frac{2x\sqrt{x^2+1}}{x^2+3x}.$$ (4 Points)

2. Let

$$f(x)=\begin{cases}\dfrac{x\sin(x-2)}{x^2-x-2}, & \text{if } x<2,\\[2ex] \dfrac{Ax+1}{x^2-1}, & \text{if } x\geq 2.\end{cases}$$

Find the value of the constant A, so that the function f is continuous at $x=2$. (4 Points)

3. Let $f(x)=\sqrt{7-3x}$. Use the definition of the derivative to find $f'(1)$. (4 Points)

4. Find $f'(x)$, if $f(x)=x^3\tan 2x+\sec(x^2+4)$. (4 Points)

5. Let $f(x)=5x^{3/5}-x+7$. Find the point at which the graph of f has a vertical tangent line. (4 Points)

6. Let $f(x)=\dfrac{|x-1|(x+3)}{(x-1)(x^2+x-6)}$. Find all points of discontinuity of f and classify each discontinuity as removable, infinite or jump. (5 Points)

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1. $\lim_{x\to -3^-} f(x) = -\infty$, $\lim_{x\to -3^+} f(x) = \infty \Longrightarrow$ $\boxed{x=-3 \text{ is VA.}}$

$$f(x) = \frac{2|x|\sqrt{1+\frac{1}{x^2}}}{x+3} : \lim_{x\to -\infty} f(x) = -2,\ \lim_{x\to \infty} f(x) = 2 \Longrightarrow \boxed{y=-2 \ \& \ y=2 \text{ are HL}}$$

2. $\lim_{x\to 2^-} f(x) = \lim_{x\to 2} \frac{x}{x+1} \lim_{x\to 2} \frac{\sin(x-2)}{(x-2)} = \frac{2}{3},\ f(2) = \frac{2A+1}{3}.$

$$\lim_{x\to 2^-} f(x) = f(2) \Longrightarrow \boxed{A = \frac{1}{2}.}$$

3. $\lim_{x\to 1} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1} \frac{\sqrt{7-3x}-2}{x-1} =$

$$\lim_{x\to 1} \frac{(7-3x)-4}{(x-1)\left(\sqrt{7-3x}+2\right)} = \lim_{x\to 1} \frac{-3}{\sqrt{7-3x}+2} = \frac{-3}{4} \Longrightarrow \boxed{f'(1) = \frac{-3}{4}.}$$

4. $f'(x) = 3x^2 \tan 2x + 2x^3 \sec^2 2x + 2x \sec(x^2+4) \tan(x^2+4)$.

5. $f'(x) = 3x^{-2/5} - 1$.

   f is continuous at $x=0$ and $\lim_{x\to 0} |f'(x)| = \infty \Longrightarrow$ the graph of f has VTL at the point $(0,7)$.

6. The discontinuities of f are at -3, 1, 2.

$$\lim f(x) = \frac{1}{\quad} \Longrightarrow f \text{ has RD at } \quad$$